GCSE Maths – Algebra

Equivalent Algebraic Expressions

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of algebra questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

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Section A

Worked Example

Categorise the following into expression, equations, and identities:

a) $x^2 + 4$ b) $x^2 + 4x = 5$ c) $4y^2 + y^2 = 5y^2$
d) $x^2\sqrt{y}$ e) $x^2 + y^2 = 25$ f) $16y^2 + 8y + 4 = 2$

Step 1: Firstly, we need to work out which ones are expressions. We know that expressions are a group of terms related to each other using mathematical operations, and do not have an equals sign.

The above definition tells us that ***a)*** *and* ***d)*** *are expressions.*

Step 2: Now, we want to see which ones are equations. An equation is a statement with an equals sign stating two expressions are equal.

Using this, we can see that b), e) and f) are equations.

Step 3: Lastly, we look for the identities. An identity is an equation that is true no matter what values are inputted.

Hence, only ***c)*** *is an identity.*

Guided Example

Categorise the following into expression, equations, and identities:

a) $xy + z = w$ b) $4x^2 + x$ c) $z^2 + 2z = 4$
d) $2(x^2 + y^2) = 2x^2 + 2y^2$ e) xyz f) $4x^2 - x^2 = 3x^2$

Step 1: Using the definition, identify which of the list are expressions.

Expressions do not have an equals sign

Therefore b, e are expressions

Step 2: Using the definition, identify which of the list are equations.

Equations are two statements with an equals sign between

Therefore a, c are equations

Step 3: Using the definition, identify which of the list are identities.

An identity is a true equation

Therefore, d, f are identities

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Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Categorise the following into expressions, equations, and identities:

a) $x + y + z$
b) $x + y + z = 64$
c) $3x + 2x - 4z + 4z^2 = 5x + 4(z^2 - z)$
d) $7x^2 + 2x + 2021 = 0$
e) $\sin x + \cos x$
f) $\tan x = \frac{\sin x}{\cos x}$

Expressions do not have an equals sign

Therefore a, e are expressions

Equations are two statements with an equals sign between

Therefore b, d are equations

An identity is a true equation

Therefore, c, f are identities

2. Write an expression that contains the information below for any number, n:

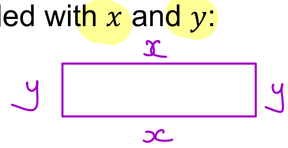
Start with a number, n, $\underbrace{\text{subtract 14}}_{①}$ and $\underbrace{\text{multiply the result by 2}}_{②}$

① $n - 14$

② $2 \times (n - 14)$

$= 2(n - 14)$

3. If the area of a rectangle is 50, and the sides are labelled with x and y:
   a) Write an equation for the area
   b) Write an expression for the perimeter



a) Area = length × width

$50 = x \times y$

$50 = xy$

b) Perimeter = Add all the sides up

$x + y + x + y$

$2x + 2y$

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Section B – Higher Only

Worked Example

Prove that the square of an odd number is odd.

Step 1: Firstly, we represent any odd number by the expression $2n+1$ where n can be any integer. Then we square it:

$$(2n+1)^2 = (2n+1)(2n+1) = 4n^2 + 2n + 2n + 1 = 4n^2 + 4n + 1$$

Step 2: Now we know that $(2n+1)^2 = 4n^2 + 4n + 1$, we need to factorise parts of it in a way that allows us to prove the statement:

Only focus on parts of the product and look for a factor that we can take out (divide terms by). In this case, we can divide the first and second terms by 4, so let's take that out as a factor.

$$4n^2 + 4n + 1 = 4(n^2 + n) + 1$$

By doing the above, we can see that $(2n+1)^2$ is the sum of an even part, $4(n^2+n)$ (since it is divisible by 4), and 1. We know that an even number plus 1 is odd, hence it is odd.

Guided Example

Prove that $(n+2)^2 - (n-2)^2$ is always divisible by 8, when n is an integer.

Step 1: Expand the brackets.

$(n+2)(n+2) = n^2 + 2n + 2n + 4 \quad = n^2 + 4n + 4$

$(n-2)(n-2) = n^2 - 2n - 2n + 4 \quad = n^2 - 4n + 4$

Step 2: Simplify the answer.

$n^2 + 4n + 4 - (n^2 - 4n + 4)$

$= \cancel{n^2} + 4n \cancel{+4} \cancel{-n^2} + 4n \cancel{-4}$

$= 8n$

Step 3: Use the simplified result to prove the required result.

$8n = 8(n)$

This is divisible by 8 as 8 is a factor.

Therefore, $(n+2)^2 - (n-2)^2$ is divisible by 8

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

4. Prove that the square of an even number is always even.

Even number $= 2n$

Square of even $= (2n)^2 = 2n \times 2n$
$= 4n^2$

$4n^2 = 2(2n^2)$

2 is a factor of the result - meaning it is even
Therefore the square of a even is always even

5. Prove that $(n+13)^2 - (n+2)^2$ is always divisible by 11, where n is any integer.

$(n+13)(n+13) = n^2 + 26n + 169$
$(n+2)(n+2) = n^2 + 4n + 4$

$n^2 + 26n + 169 - (n^2 + 4n + 4)$
$= n^2 + 26n + 169 - n^2 - 4n - 4$
$= 22n + 165$
$= 11(2n + 15)$

11 is a factor of the result.
Therefore $(n+13)^2 - (n+2)^2$ is always divisible by 11.

6. Prove that the sum of three consecutive numbers is always divisible by 3.

Consecutive numbers: $n-1, n, n+1$

Sum: $(n-1) + n + (n+1)$
$= n \cancel{-1} + n + n \cancel{+1}$
$= 3n$
$= 3(n)$

3 is a factor of the result.
Therefore, the sum of 3 consecutive numbers is divisible by 3.

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